NACS 643 Lab #1
Spring 2009

This lab is a modified version of some of the optional questions from Homework 4.

Fort this lab, we'll use a different method of integrating (i.e. solving) the differential equations. This method is more powerful than the one used in the homework. It was presented in lecture and is also in Appendix A of Chapter 5 in the text.

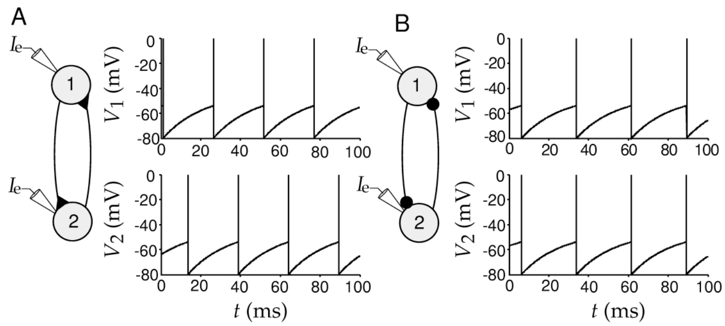
A) We'll start with an equation in the form $\tau_u \frac{du}{dt} = -u + u_\infty(t)$. For instance, τ_u is a time constant, u is the variable of interest (e.g. voltage), and $u_\infty(t)$ is everything else, e.g. constant terms proportional to conductance and reversal potentials, but also time varying terms such as external currents and synaptic conductances.
B) Choose a small time step Δt (much smaller than τ) so we can make time discrete: $t_k = k\Delta t$, and we'll evaluate $u(t)$ only at those discrete times: $u(t_k) = u_k$ and $u(t_k + \Delta t) = u(t_{k+1}) = u_{k+1}$.
C) The goal is to generate u at each future time step (k+1) based on its value (and the value of the external input) at the current time step (k), and do it in a way that is extremely stable against numerical artifacts. That is, we calculate u_{k+1} from u_k, $u_\infty(t_k) = u_{\infty k}$, Δt, and the other constants.
D) We note that as long as $u_\infty(t)$ changes very little over the course of a single Δt, we get $u(t+\Delta t) \approx u_\infty(t) + (u(t) - u_\infty(t))\exp(-\Delta t / \tau_u)$, since that's the exact (exponential) solution in the case $u_\infty(t)$ being a constant.
E) We implement this exactly, which turns out to be correct with error proportional to Δt, using $u_{k+1} = u_{\infty k} + (u_k - u_{\infty k})\exp(-\Delta t / \tau_u)$, which solves the equation in (A), one time step at a time.

1) Build a model integrate-and-fire neuron using the equation $\tau_m \frac{dV}{dt} = -V + E_L - r_m \bar{g}_s P_s (V - E_s) + R_m I_e$. To put this into the form used above, you'll need to rewrite it as:
$\frac{\tau_m}{1 + r_m \bar{g}_s P_s} \frac{dV}{dt} = -V + \frac{E_L + r_m \bar{g}_s P_s E_s + R_m I_e}{1 + r_m \bar{g}_s P_s}$. Use $\tau_\mathrm{m} = 10$ ms. Initially set V to $V_\mathrm{rest} = E_L = -70$ mV.
When the membrane potential reaches $V_\mathrm{th} = -54$ mV, make the neuron fire a spike and reset the potential to $V_\mathrm{reset} = -80$ mV. Using $E_s = 0$, set the external current to zero, $I_e = 0$, in this example, and $r_m \bar{g}_s = 0.5$. Assume that the probability of release on receipt of a presynaptic spike is 1. To *correctly* incorporate multiple presynaptic spikes, $P_s(t)$ should be described by a pair of differential equations,

$$\tau_s \frac{dP_s}{dt} = -P_s + eP_\mathrm{max} z \text{ and } \tau_s \frac{dz}{dt} = -z$$

where $e = \exp(1)$, with the additional rule that z is set to 1 whenever a presynaptic spike arrives. (This means that P_s will be an alpha function: $P_s \propto t\exp(-t/\tau_s)$ whenever a single synapse is active, but not when more than one are). Use $\tau_s = 10$ ms and $P_\mathrm{max} = 0.5$.

Trigger synaptic events at times 50, 150, 190, 300, 320, 400 and 410 ms. In separate graphs, plot $z(t)$ (called the conductance initiator), $P_s(t)$, $V(t)$ and the synaptic current (or, more simply, $r_m\overline{g}_s P_s(V-E_s)$). Explain what you see in each plot. How which of the seven synaptic conductances leads to a spike?



2) Construct a model of two coupled integrate-and-fire neurons similar to that in figure 5.20 of Dayan & Abbot (and shown above). Both model neurons obey $\tau_m \dfrac{dV}{dt} = -V + E_L - r_m\overline{g}_s P_s(V-E_s) + R_m I_e$ with $E_L = -70$ mV, $V_{\text{th}} = -54$ mV, $V_{\text{reset}} = -80$ mV, $\tau_{\text{m}} = 20$ ms $r_m\overline{g}_s = 0.15$, and $R_m I_e = 18$ mV. Both synapses should be described as in problem 2 with $P_{\text{max}} = 0.5$ and $\tau_s = 5$ ms. Consider cases where both synapses are excitatory, with $E_s = 0$, and both are inhibitory, with $E_s = -80$. To use the integration technique above, you'll want to rewrite the differential equation as:

$$\frac{\tau_m}{1 + r_m\overline{g}_s P_s}\frac{dV}{dt} = -V + \frac{E_L + r_m\overline{g}_s P_s E_s + R_m I_e}{1 + r_m\overline{g}_s P_s}.$$

a) Show how the pattern of firing for the two neurons depends on the type (excitatory or inhibitory) of the reciprocal synaptic connections. For these simulations, set the initial membrane voltages of the two neurons to slightly different values, randomly, and run the simulation until an equilibrium situation has been reached, which may take a few seconds of simulated run time. Start from a few different random initial conditions to study whether the results are consistent. Plot for both cases, $P_s(t)$ and $V(t)$ for the last 500 ms of your simulation.

b) Optional: Investigate what happens if you change the strengths and time constants of the synapses.